

In Exercises 13–18, state the domain and range of the function and sketch a graph showing four periods.

13. $y = \cos^2 x$ 14. $y = |\cos x|$
 15. $y = |\cot x|$ 16. $y = \cos |x|$
 17. $y = -\tan^2 x$ 18. $y = -\sin^2 x$

The graph of each function in Exercises 19–22 oscillates between two parallel lines, as in Example 5. Find the equations of the two lines and graph the lines and the function in the same viewing window.

19. $y = 2x + \cos x$ 20. $y = 1 - 0.5x + \cos 2x$
 21. $y = 2 - 0.3x + \cos x$ 22. $y = 1 + x + \cos 3x$

In Exercises 23–28, determine whether $f(x)$ is a sinusoid.

23. $f(x) = \sin x - 3 \cos x$ 24. $f(x) = 4 \cos x + 2 \sin x$
 25. $f(x) = 2 \cos \pi x + \sin \pi x$ 26. $f(x) = 2 \sin x - \tan x$
 27. $f(x) = 3 \sin 2x - 5 \cos x$ 28. $f(x) = \pi \sin 3x - 4\pi \sin 2x$

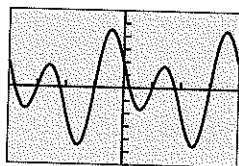
In Exercises 29–34, find a , b , and h so that $f(x) \approx a \sin(b(x-h))$.

29. $f(x) = 2 \sin 2x - 3 \cos 2x$ 30. $f(x) = \cos 3x + 2 \sin 3x$
 31. $f(x) = \sin \pi x - 2 \cos \pi x$ 32. $f(x) = \cos 2\pi x + 3 \sin 2\pi x$
 33. $f(x) = 2 \cos x + \sin x$ 34. $f(x) = 3 \sin 2x - \cos 2x$

In Exercises 35–38, the function is periodic but not a sinusoid. Find the period graphically and sketch a graph showing one period.

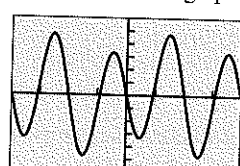
35. $y = 2 \cos x + \cos 3x$ 36. $y = 2 \sin 2x + \cos 3x$
 37. $y = \cos 3x - 4 \sin 2x$ 38. $y = \sin 2x + \sin 5x$

In Exercises 39–42, match the function with its graph.



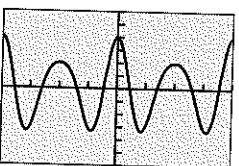
$[-2\pi, 2\pi]$ by $[-6, 6]$

(a)



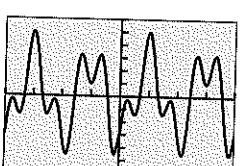
$[-2\pi, 2\pi]$ by $[-6, 6]$

(b)



$[-2\pi, 2\pi]$ by $[-6, 6]$

(c)



$[-2\pi, 2\pi]$ by $[-6, 6]$

(d)

39. $y = 2 \cos x - 3 \sin 2x$ 40. $y = 2 \sin 5x - 3 \cos 2x$
 41. $y = 3 \cos 2x + \cos 3x$ 42. $y = \sin x - 4 \sin 2x$

In Exercises 43–48, tell whether the function exhibits damped oscillation. If so, identify the damping factor and tell whether the damping occurs as $x \rightarrow 0$ or as $x \rightarrow \infty$.

43. $f(x) = e^{-x} \sin 3x$ 44. $f(x) = x \sin 4x$
 45. $f(x) = \sqrt{5} \cos 1.2x$ 46. $f(x) = \pi^2 \cos \pi x$

47. $f(x) = x^3 \sin 5x$

48. $f(x) = \left(\frac{2}{3}\right)^x \sin\left(\frac{2x}{3}\right)$

In Exercises 49–52, graph both f and plus or minus its damping factor in the same viewing window. Describe the behavior of the function f for $x > 0$. What is the end behavior of f ?

49. $f(x) = 1.2^{-x} \cos 2x$ 50. $f(x) = 2^{-x} \sin 4x$
 51. $f(x) = x^{-1} \sin 3x$ 52. $f(x) = e^{-x} \cos 3x$

In Exercises 53–56, find the period and graph the function over two periods.

53. $y = \sin 3x + 2 \cos 2x$
 54. $y = 4 \cos 2x - 2 \cos(3x - 1)$
 55. $y = 2 \sin(3x + 1) - \cos(5x - 1)$
 56. $y = 3 \cos(2x - 1) - 4 \sin(3x - 2)$

In Exercises 57–62, graph f over the interval $[-4\pi, 4\pi]$. Determine whether the function is periodic and, if it is, state the period.

57. $f(x) = \left|\sin \frac{1}{2}x\right| + 2$ 58. $f(x) = 3x + 4 \sin 2x$
 59. $f(x) = x - \cos x$ 60. $f(x) = x + \sin 2x$
 61. $f(x) = \frac{1}{2}x + \cos 2x$ 62. $f(x) = 3 - x + \sin 3x$

In Exercises 63–70, find the domain and range of the function.

63. $f(x) = 2x + \cos x$ 64. $f(x) = 2 - x + \sin x$
 65. $f(x) = |x| + \cos x$ 66. $f(x) = -2x + |3 \sin x|$
 67. $f(x) = \sqrt{\sin x}$ 68. $f(x) = \sin |x|$
 69. $f(x) = \sqrt{|\sin x|}$ 70. $f(x) = \sqrt{\cos x}$

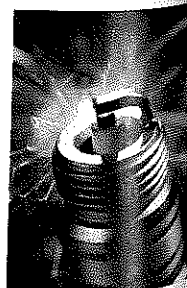
71. **Oscillating Spring** The oscillations of a spring subject to friction are modeled by the equation $y = 0.43e^{-0.55t} \cos 1.8t$.

- (a) Graph y and its two damping curves in the same viewing window for $0 \leq t \leq 12$.
 (b) Approximately how long does it take for the spring to be damped so that $-0.2 \leq y \leq 0.2$?

72. **Predicting Economic Growth** The business manager of a small manufacturing company finds that she can model the company's annual growth as roughly exponential, but with cyclical fluctuations. She uses the function $S(t) = 75(1.04)^t + 4 \sin(\pi t/3)$ to estimate sales (in millions of dollars), t years after 2005.

- (a) What are the company's sales in 2005?
 (b) What is the approximate annual growth rate?
 (c) What does the model predict for sales in 2013?
 (d) How many years are in each economic cycle for this company?

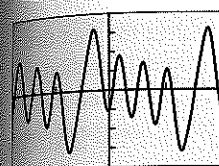
73. **Writing to Learn** Example 3 shows that the function $y = \sin^2 x$ is periodic. Explain whether you think that $y = \sin x^3$ is periodic and why.



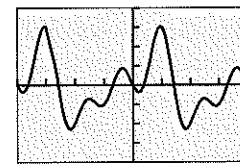
In Exercises 75 and 76, select the one correct inequality, (a) or (b). Give a convincing argument.

75. (a) $x - 1 \leq x + \sin x \leq x + 1$ for all x .
 (b) $x - \sin x \leq x + \sin x$ for all x .
 76. (a) $-x \leq x \sin x \leq x$ for all x .
 (b) $-|x| \leq x \sin x \leq |x|$ for all x .

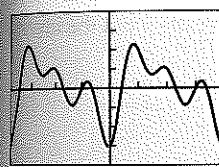
In Exercises 77–80, match the function with its graph. In each case state the viewing window.



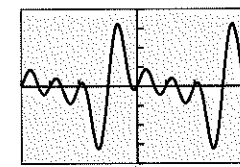
(a)



(b)



(c)



(d)

77. $y = \cos x - \sin 2x - \cos 3x + \sin 4x$
 78. $y = \cos x - \sin 2x - \cos 3x + \sin 4x - \cos 5x$
 79. $y = \sin x + \cos x - \cos 2x - \sin 3x$
 80. $y = \sin x - \cos x - \cos 2x - \cos 3x$

Standardized Test Questions

81. **True or False** The function $f(x) = \sin |x|$ is periodic. Justify your answer.
 82. **True or False** The sum of two sinusoids is a sinusoid. Justify your answer.

You may use a graphing calculator when answering these questions.

83. **Multiple Choice** What is the period of the function $f(x) = |\sin x|$?
 (A) $\pi/2$ (B) π (C) 2π
 (D) 3π (E) None; the function is not periodic.
 84. **Multiple Choice** The function $f(x) = x \sin x$ is
 (A) discontinuous. (B) bounded. (C) even.
 (D) one-to-one. (E) periodic.
 85. **Multiple Choice** The function $f(x) = x + \sin x$ is
 (A) discontinuous. (B) bounded. (C) even.
 (D) odd. (E) periodic.

86. **Multiple Choice** Which of the following functions is *not* a sinusoid?

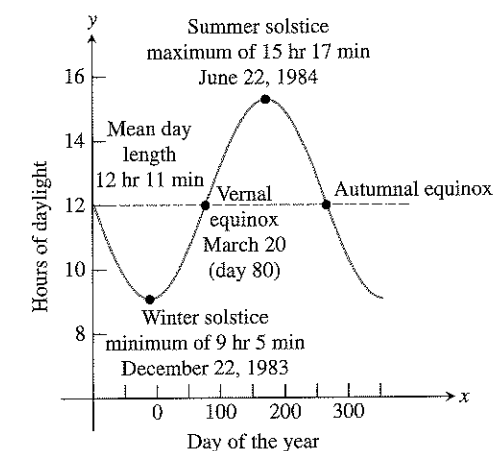
- (A) $2 \cos(2x)$ (B) $3 \sin(2x)$ (C) $3 \sin(2x) + 2 \cos(2x)$
 (D) $3 \sin(3x) + 2 \cos(2x)$ (E) $\sin(3x + 3) + \cos(3x + 2)$

Explorations

87. **Group Activity Inaccurate or Misleading Graphs**

- (a) Set Xmin = 0 and Xmax = 2π . Move the cursor along the x -axis. What is the distance between one pixel and the next (to the nearest hundredth)?
 (b) What is the period of $f(x) = \sin 250x$? Consider that the period is the length of one full cycle of the graph. Approximately how many cycles should there be between two adjacent pixels? Can your grapher produce an accurate graph of this function between 0 and 2π ?

88. **Group Activity Length of Days** The graph shows the number of hours of daylight in Boston as a function of the day of the year, from September 21, 1983, to December 15, 1984. Key points are labeled and other critical information is provided. Write a formula for the sinusoidal function and check it by graphing.



Extending the Ideas

In Exercises 89–96, first try to predict what the graph will look like (without too much effort, that is, just for fun). Then graph the function in one or more viewing windows to determine the main features of the graph, and draw a summary sketch. Where applicable, name the period, amplitude, domain, range, asymptotes, and zeros.

89. $f(x) = \cos e^x$ 90. $g(x) = e^{\tan x}$
 91. $f(x) = \sqrt{x} \sin x$ 92. $g(x) = \sin \pi x + \sqrt{4 - x^2}$
 93. $f(x) = \frac{\sin x}{x}$ 94. $g(x) = \frac{\sin x}{x^2}$
 95. $f(x) = x \sin \frac{1}{x}$ 96. $g(x) = x^2 \sin \frac{1}{x}$